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A GRAPHICAL METHOD FOR ANALYZING POOL-BOILING SYSTEMS

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NOMENCLATURE

- A, surface area [m^2];
- b, temperature coefficient of electrical resistance [K^{-1}];
- c, specific heat capacity [$J\,kg^{-1}\,K^{-1}$];
- d, diameter [m];
- e, base of natural logarithms (2·718 . . .);
- i, electrical current [A];
- k, coefficient of heat transfer between convective fluid and boiling surface [$W\,m^{-2}\,K^{-1}$];
- p, perimeter of heating element [m];
- q, heat flux density [$W\,m^{-2}$];
- $\dot{q}$, heat generation per unit surface [$W\,m^{-2}$];
- S, cross-sectional area [m^2];
- t, temperature averaged with respect to space and time [K];
- t', temperature local and instantaneous [K];
- V, volume [m^3];
- x, longitudinal distance from end of boiling zone [m].

Greek letters

- α, surface coefficient of convective heat transfer [$W\,m^{-2}\,K^{-1}$];
- Δ, finite difference;
- θ, time [s];
- λ, thermal conductivity [$W\,m^{-1}\,K^{-1}$];
- ρ, density [$kg\,m^{-3}$];
- $\bar{\rho}$, specific electrical resistance [$\Omega\,m$];
- ϑ, excess temperature, equation (1) [K].

Subscripts

- c, convective fluid;
- f, quantities for film boiling;
- i, inside of circular tube;
- l, quantities for saturated liquid;
- n, quantities for nucleate boiling;
- o, outside circular tube;
- s, quantities at saturation conditions;
- v, quantities for saturated vapor;
- 0, initial condition;
- x, intercept on ϑ-axis

INTRODUCTION

THE LITERATURE on boiling heat transfer abounds in figures indicating the relationship between the heat flux density q, and the excess temperature of the boiling surface ϑ, defined by the equation

$$\vartheta = t - t_s. \tag{1}$$

These curves are frequently very qualitative and usually they are plotted as log q against log ϑ for compactness and convenience, and their essential use is to indicate the extent of the three boiling regimes, nucleate, transition, and stable film boiling.

In the following it will be shown, that plotting with linear scales and to simultaneously show operating lines (or curves), provide much more insight in the physical phenomena of pool boiling, and that such a graphical solution has great usefulness for studying and designing pool boiling experiments.

1. THE GRAPHICAL METHOD

1.1 *Characteristic boiling curves*

Implicit in the representation of a boiling curve is the assumption that to maintain a steady-state heat flux from a surface by boiling heat transfer, it is a necessary and sufficient condition to maintain an excess temperature, ϑ, between the surface and the bulk of the boiling fluid, the latter usually being taken as the saturation temperature, t_s, corresponding to the prevailing pressure.

The instantaneous value of ϑ determines uniquely the heat flux density for a given surface and fluid at a fixed pressure.

This statement implies that the previous history of the system is not reflected in its present state: this cannot be

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absolutely true, and a boiling system must to some extent be affected by the immediately preceeding state. In the following this effect will be considered negligible.

Such characteristic boiling curves have been determined experimentally by Nukijama, Farber and Scorah, McAdams *et al.* [1–3], and by many others. As an example, for water, see (Fig. 1).

It must be kept clearly in mind that the surface temperature, used in calculating ϑ, is an average with respect to space as well as time.

$$t = \frac{1}{A\theta}\int_{A}^{A+dA}\int_{0}^{\theta} t'\,dA\,d\theta. \qquad (2)$$

This statement is true for all three boiling regimes, the nucleate until the first boiling crisis I on Fig. 1, the transition between the first and second boiling crisis I and II, and the stable film boiling regime beyond II.

heat-flux density, taking into account all the variables that determine the surface temperature as a function of the heat flux density. For a given system, the only possible states are those indicated by points common to the two curves; the boiling curve, and the operating curve. For the present the operating curves will be represented approximately as straight lines (operating lines).

2. APPLICATIONS

2.1 *Determination of boiling curves*

Several different methods have been used for determination of the boiling curve they are basically: quenching, convective heating, and electrical heating.

Each method has its own special advantages and disadvantages which will appear during the subsequent analysis.

2.1.1 *Quenching of a small body.* This is a quasi-steady-state

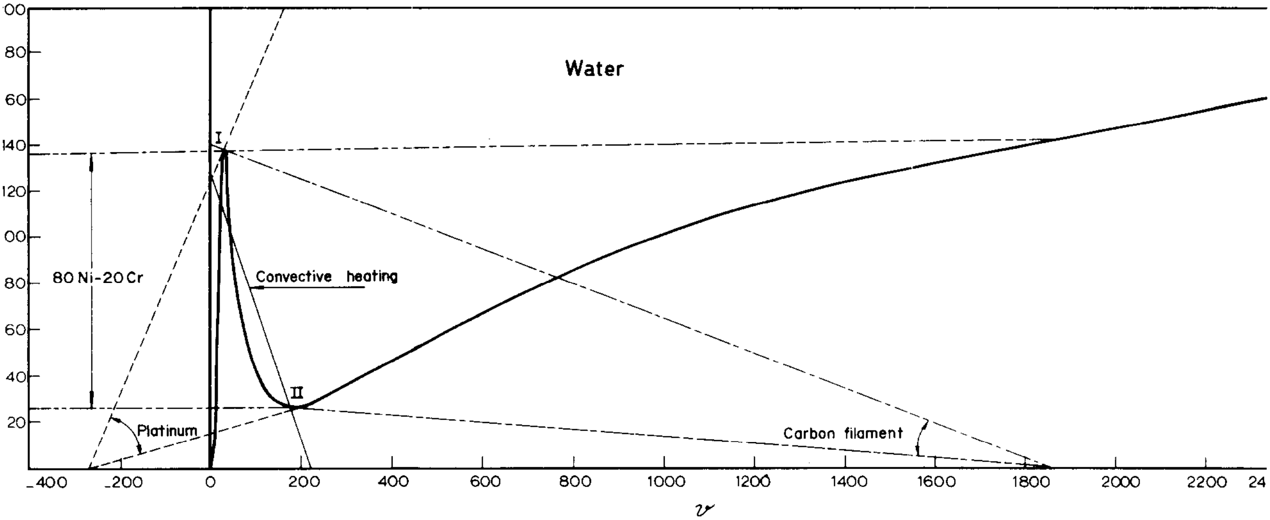

FIG. 1. Approximate boiling curve for water boiling on a wire or a horizontal surface at atmospheric pressure.

Every point on (Fig. 1) indicates a possible steady-state heat flux and its corresponding heat flux density. However, the actual conditions in a physical system will depend on the means used for attaining the heat flux as well. That is the actual heat flux will depend on the previous states, the thermal properties of the surface, and if electrically heated on the electrical properties of the heating element and the electrical circuit used for supplying the power.

1.2 *Operating curves*

Another curve, the operating curve, may be drawn on the same diagram as the boiling curve. This curve will express the relationship between the surface temperature and the

method and if a unique boiling curve is to be obtained it must be assumed that the instantaneous relationship between θ and q is not affected by the change of temperature. Since there is no restraint on the temperature of the body it has been eliminated by the introduction of another variable, time and the operating line degenerates to a point on the boiling curve. Observation of the temperature at one or more points of the body, allows estimation of the surface temperature, and the rate of change of temperature yields the heat flux to the boiling fluid.

From a curve or curves showing the temperature as a function of time both ϑ and q may be calculated and a boiling curve drawn. If such an experiment be started on the

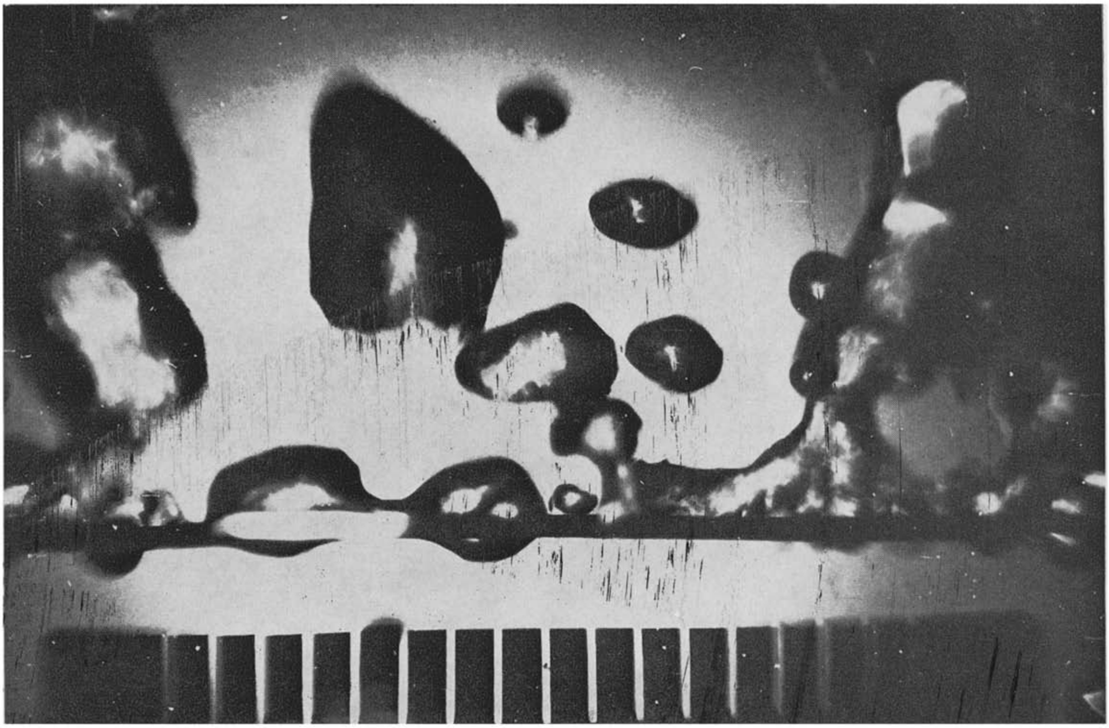

FIG. 2. Coexistent boiling on a platinum wire.

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film boiling curve the whole characteristic boiling curve may be traced out. This may or may not be the same curve as found for the steady state, most likely this would depend on the rate of cooling.

2.1.2 *Convectively heated tube (boiling on outside).* For such a tube the outside surface temperature is a function of the temperature of the convective fluid and the thermal resistance between the tube surface and the fluid. This relationship may be expressed by the well-known heat transfer equation

$$\vartheta_c - \vartheta = t_c - (t_s + \vartheta) = \frac{q}{\dfrac{d_o/d_i}{\alpha_i} + \dfrac{d_o \ln d_o/d_i}{\lambda}} \tag{3}$$

or substituting

$$k = \frac{d_o/d_i}{\alpha_i} + \frac{d_o \ln d_o/d_i}{\lambda}$$

and rearranging

$$\vartheta = \vartheta_c - q/k \tag{4}$$

that is by a straight line on the boiling diagram with a slope $-k$ and intersecting the ϑ-axis at $\vartheta = \vartheta_c$.

The operating line shown on (Fig. 1) is for a copper tube with steam condensing at 75×10^5 N/m^2.

To obtain q as a single valued function of ϑ on the transition branch of the boiling curve, it is evident from (Fig. 1) that the negative slope of the operating line must be greater than that of the boiling curve $-k > (-\mathrm{d}q/\mathrm{d}\vartheta)$. Therefore, to obtain data for the transition branch would require equipment with a very high value for k, this is unobtainable with water using ordinary techniques for heat transfer. on the other hand it should be possible to have coexistent nucleate, transition, and stable film boiling on a convectively heated tube if $-k < (-\mathrm{d}q/\mathrm{d}\vartheta)$. To reach such a state the temperature would have to be increased until film boiling occur locally, and then the tube be cooled to an appropriate intermediate temperature where the operating line would intersect the boiling curve in three places.

With certain fluids the negative slope of the transition curve is so low that a single root is easily obtained.

2.1.3 *Electrically heated thin wire.* If the current in a wire is considered to be the independent variable, a thermally thin wire will act as a linear heat source of a local strength proportional to $\bar{\rho}_s$, the specific electrical resistance. The power density based on unit surface area will be

$$\dot{q} = \frac{\bar{\rho}_s I^2 \,\mathrm{d}x}{S} / P\mathrm{d}x = \frac{\bar{\rho}_s I^2}{PS} \tag{5}$$

as a first approximation (accurate enough for our purpose), the specific resistance may be written [5]

$$\bar{\rho} = \bar{\rho}_s (1 + b\vartheta) \tag{6}$$

substituting this value in equation (3)

$$\dot{q} = \frac{\bar{\rho}_s (1 + b\vartheta)}{PS} I^2 \tag{7}$$

and again we have a straight operating line. This type of line is shown on (Fig. 1) for a pure metal, platinum, an alloy 80 Cr–20 Ni, and for a carbon filament. For each material the operating line will pass through a point on the ϑ axis,

$$\vartheta_x = -1/b \tag{8}$$

for a pure metal this point of intersection is usually near the absolute zero, for most alloys it is much lower and for a carbon filament it is near 2400 K.

It should be clear from the figure (Fig. 1) that the actual roots existing with a particular kind of heating depends on the history of the system. If the heating starts with a cold element the only real root, that can satisfy the conditions, will be on the nucleate boiling branch of the curve, the only way that stable film boiling and roots on the transition curve may be obtained is by the operating line departing from the tangent point on the boiling curve. If this happens with a pure metal it can be seen that the heat generation would increase very rapidly locally and burnout would occur unless the wire has a very high melting point or that the spreading of the film boiling zone is very rapid. Since the spreading of the film-boiling zone (a thermal process) will increase the resistance of a pure metal considerably, it is possible that the current in the circuit will drop to a value so low, that burnout will not occur; that is, if the circuit is properly designed.

This lower current value may of course also be produced increasing the external resistance of the circuit, if it occurs rapidly enough, and if the total resistance of the circuit is properly adjusted, it is possible to have a zone of nucleate boiling, a zone covering all intermediate temperatures, and a zone of stable film boiling. A photograph of such an occurrence as observed by van Stralen[6] is shown on Fig. 2.

2.2 *Coexistent boiling*

2.2.1 *The differential equation for coexistent boiling.* If we assume; steady state, that the material from which the heating element is made is homogeneous, and that end effects may be neglected, the energy equation for a differential element (Fig. 3) may be written as:

Rate of heat flow by conduction into element at $x + \mathrm{d}x$	+	Rate of heat generation by sources within element	−	Rate of heat flow by conduction out of element at x	−	Rate of heat flow by boiling from surface between x and $x + \mathrm{d}x$	=	Accumulation of heat in the element between x and $x + \mathrm{d}x$

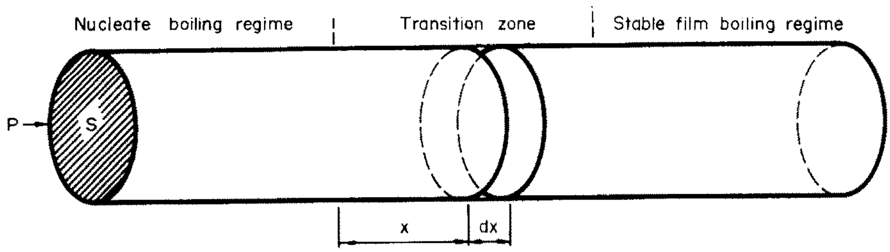

FIG. 3. A sketch of physical quantities used in writing the differential equation (10).

or expressed in mathematical form using well known conduction and convection heat transfer equations.

$$\left[-\lambda S\frac{\partial t}{\partial x}+\frac{\partial}{\partial x}\left(-\lambda S\frac{\partial t}{\partial x}\right)\mathrm{d}x\right]+\left[q\,P\mathrm{d}x\right]+\left[\lambda S\frac{\partial t}{\partial x}\right]-\left[P\mathrm{d}x\right]=c\rho\frac{\partial t}{\partial \theta}S\mathrm{d}x \tag{9}$$

or after reduction

$$\frac{\partial^2 t}{\partial x^2}-P/S(q-\dot{q})=c\rho\frac{\partial t}{\partial\theta} \tag{10}$$

or for steady-state conditions, when $\partial t/\partial\theta = 0$

$$\frac{\mathrm{d}^2 t}{dx^2}=\frac{P/S}{\lambda}(q-\dot{q}). \tag{11}$$

2.2.2 *Steady state.* Equation (11) may be solved by first substituting $\mathrm{d}t/\mathrm{d}x = y$

$$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{P/S}{\lambda}(q-\dot{q}) \tag{12}$$

then dividing the left side of equation (12) by $\mathrm{d}t/\mathrm{d}x$, and the right by y

$$\frac{\mathrm{d}y}{\mathrm{d}t}=\frac{P/S}{y\lambda}(q-\dot{q}) \tag{13}$$

separating variables, integrating between the limits $y = 0$ when $t = t_n$, and $y = y$ when $t = t$, and dropping the minus before the square root sign as having no physical significance

$$\frac{\mathrm{d}t}{dx}=y=\left[\frac{2P}{S}\int_{t_n}^{t}\frac{1}{\lambda}(q-\dot{q})\,\mathrm{d}t\right]^{\frac{1}{2}} \tag{14}$$

there is still another boundary condition to satisfy for steady state, viz. $y = 0$ at $t = t_f$, therefore

$$\int_{t_n}^{t_f}\frac{1}{\lambda}(q-\dot{q})\,\mathrm{d}t=0. \tag{15}$$

This last equation may be solved numerically or graphically and when the operating line has been located in this manner, equation (14) may be used for finding $\mathrm{d}t/\mathrm{d}x$ as a function of x, and t as a function of x by subsequent integration of this equation.

2.2.3 *Unsteady state.* Assuming that the time constant for the response of the electrical system is very short compared to the time constant for the thermal response, equation (7) may be differentiated holding everything but $\dot{q}$ and I constant, and equation (10) differentiated holding everything but $\partial t/\partial\theta$ and $\dot{q}$ constant

$$\partial\left(\frac{\partial t}{\partial\theta}\right)=\frac{2\bar{\rho}_s(1+b\vartheta)I}{S^2c\rho}\partial I \tag{16}$$

for a small increase in the current, a step function ΔI, maintained for a short interval and starting from the steady state location of the operating line where $\partial t/\partial\theta = 0$ we can write

$$\frac{\partial t}{\partial\theta}=\frac{2\bar{\rho}_s(1+b\vartheta)I}{S^2c\rho}\Delta I \tag{17}$$

or since $\partial t = \partial$, by differentiation of equation (17)

$$\frac{\partial(1+b\vartheta)}{\partial\theta(1+b\vartheta_0)}=\frac{2b\bar{\rho}_s I}{S^2c\rho}\Delta I \tag{18}$$

or integrating from $\theta = 0$ when $\vartheta = \vartheta_0$ to $\theta = \theta$ where $\vartheta = \vartheta$

$$\ln\frac{1+b\vartheta}{1+b\vartheta_0}=\frac{2b\bar{\rho}_s I}{S^2c\rho}(\Delta I\theta) \tag{19}$$

or solving for $\Delta t = \vartheta - \vartheta_0$

$$\Delta t=\vartheta-\vartheta_0=\left(\vartheta_0+\frac{1}{b}\right)\left(\exp\frac{b\bar{\rho}_s I}{S^2c\rho}\right)(\Delta I\theta) \tag{20}$$

this equation indicates that the instantaneous change of temperature increases in absolute value with ϑ_0, and is negative when

$$-\frac{1}{\vartheta_0}<b<0, \tag{21}$$

when $b = 0$, the value of Δt is

$$\Delta t = \frac{\bar{\rho}_s I}{S^2 c\rho}(\Delta I\theta) \tag{22}$$

in this case Δt will be uniform over the length of the wire and there will be no longitudinal heat flux. When the change in Δt is positive there will be a longitudinal heat flux from high ϑ_0 towards low ϑ_0, that is from the film boiling zone towards the nucleate boiling zone, when b is negative the flux will be in the other direction from the nucleate toward the film boiling zone, when $b = 0$ there will be no tendency from the zones to change in extent.

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LEADING EDGE EFFECTS ON THE NUSSELT NUMBER FOR A VERTICAL PLATE IN FREE CONVECTION

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NOMENCLATURE

C_p, specific heat of the fluid;
g, acceleration due to gravity;
Gr_x, local Grashof number $\rho^2\beta g(T_w - T_\infty)\, x^3/\mu^2$;
h_c, free convective coefficient;
k, thermal conductivity of the fluid;
L, width of the plate;
Nu_x, local Nusselt number $h_c x/k$;
Pr, Prandtl number $C_p\mu/k$;
q, heat transfer per unit area;
T, temperature;
ΔT, temperature difference;
ΔN, fringe shift;
n, refractive index of fluid;
x, distance along the length of the plate;
y, distance perpendicular to the plate.

Greek symbols
β, coefficient of thermal expansion of the fluid;
λ, wavelength of light;
μ, absolute viscosity of the fluid;
ρ, density of the fluid.

Subscripts
w, wall conditions;
∞, undisturbed fluid;
L, based on overall height of the plate.

INTRODUCTION

It is to be expected that the classical boundary layer solution by Schmidt and Beckmann [1] for local heat transfer rates from a vertical plate in free convection

$$Nu_x = 0{\cdot}360\,(Gr_x)^{\frac{1}{4}} \tag{1}$$

and the integral method by Eckert and Drake [2] which yielded

$$Nu_x = 0{\cdot}378\,(Gr_x)^{\frac{1}{4}} \tag{2}$$

are not adequate to explain the phenomenon of free convection from a vertical plate, near the leading edge, since